Robust Iterative Learning Control with Experimental Validation

Tong Duy Son, Goele Pipeleers and Jan Swevers
Division PMA, Department of Mechanical Engineering,
Katholieke Universiteit Leuven, Celestijnenlaan 300B, B-3001 Heverlee, Belgium
Email: tong.duyson@mech.kuleuven.be

1 Introduction

Iterative learning control (ILC) is widely used in control applications to improve performance of repetitive processes [1]. The key idea of ILC is to update the control signal iteratively based on measured data from previous trials, such that the output converges to the given reference trajectory. Most ILC update laws use the system model as a basis for the learning algorithm and the convergence analysis. Since system models are never perfect in practice, accounting for model uncertainty in the ILC design needs to be addressed.

This paper presents an approach to deal with model uncertainty in norm-optimal ILC. The robust ILC input is computed by minimizing the worst-case value of a performance index under model uncertainty, yielding a convex optimization problem. The proposed robust design is experimentally validated on a lab scale overhead crane system, showing the advantages of the approach over classical ILC in monotonic convergence and tracking performance.

2 System Representation

We consider an LTI, single-input single-output (SISO) system that is subject to unstructured additive uncertainty, $P_\Delta(q) = \hat{P}(q) + \Delta(q)W(q)$ and $\Delta(q) \in \mathscr{B}_\Delta$ with

$$\mathscr{B}_\Delta = \{\Delta(q) = \text{stable, causal LTI system} : \|\Delta(q)\|_\infty \leq 1\},$$

where $\|.\|_\infty$ is the $\mathscr{H}_\infty$ norm. $\hat{P}(q)$ is the nominal plant model and the weight $W(q)$ determines the size of the uncertainty. $\hat{P}(q)$, $W(q)$, and $\Delta(q)$ are stable transfer functions.

The ILC design is formulated in the trial domain, relying on the lifted system representation. The system is then represented by: $\mathbf{P}_\mathbf{\Delta} = \hat{\mathbf{P}} + \mathbf{\Delta}\mathbf{W}$, where $\hat{\mathbf{P}}, \mathbf{W}$ and $\mathbf{\Delta}$ are lower triangular Toeplitz matrices. Moreover, in order to obtain a tractable reformulation of the proposed robust ILC design, the set $\mathscr{B}_\Delta$ is replaced by an outer approximation:

$$\mathscr{B}^o_\mathbf{\Delta} = \left\{\mathbf{\Delta} \in \mathbb{R}^{N\times N} : \|\mathbf{\Delta}\| \leq 1\right\},$$

where $\|.\|$ is the induced matrix 2-norm.

3 Robust ILC Design

In norm-optimal ILC, the control signal is computed by minimizing the following cost function with respect to $\mathbf{u}_{j+1}$:

$$J(\mathbf{u}_{j+1}, \mathbf{\Delta}) = \|\mathbf{e}_{j+1}\|^2_\mathbf{Q} + \|\mathbf{u}_{j+1} - \mathbf{u}_j\|^2_\mathbf{R} + \|\mathbf{u}_{j+1}\|^2_\mathbf{S},$$

where $\mathbf{e}_{j+1}$ is the $(j+1)$-th trial tracking error, and is given by $\mathbf{e}_{j+1} = \mathbf{e}_j - (\hat{\mathbf{P}} + \mathbf{\Delta}\mathbf{W})(\mathbf{u}_{j+1} - \mathbf{u}_j)$. Note that classical norm-optimal ILC assumes $\mathbf{\Delta} = 0$ in the cost function.

We propose a robust norm-optimal ILC design by considering the following worst-case optimization problem:

$$\underset{\mathbf{u}_{j+1}}{\text{minimize}} \sup_{\|\mathbf{\Delta}\|\leq 1} \left\{J\left(\mathbf{u}_{j+1}, \mathbf{\Delta}\right)\right\}.$$

Relying on Lagrange duality, this problem is reformulated as a convex optimization problem.

The proposed robust ILC design is experimentally validated and compared with classical norm-optimal ILC and zero-phase low-pass filter ILC on a lab scale overhead crane. The experimental results (see Figure 1) show that the classical ILC yields divergence of tracking error while the robust ILC achieves monotonic convergence. Adding a zero-phase low-pass filter helps the classical ILC to avoid divergence, however at the cost of larger converged tracking error [2].

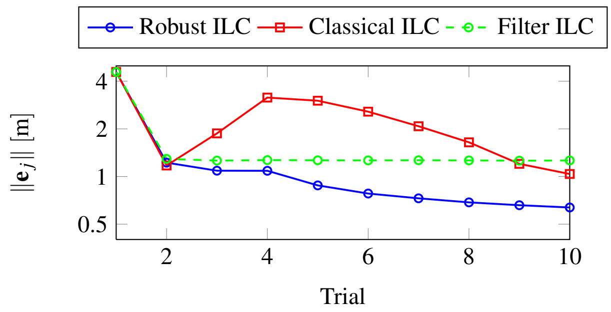

Figure 1: Tracking errors in trial domain of ILC controllers

Acknowledgements

This work was supported by the European Commission under the EU Framework 7 funded Marie Curie Initial Training Network (ITN) IMESCON (grant no. 264672). This work also benefits from the projects IWT-SBO 80032 (LECOPRO) of IWT-Vlaanderen and G.0377.09 of FWO-Vlaanderen; from KU Leuven-BOF PFV/10/002 Cente of Excellence: Optimization in Engineering (OPTEC); and from the Belgian Network Dynamical Systems, Control and Optimization (DYSCO), initiated by the Belgian Science Policy Office.

References

[1] D.A. Bristow, M. Tharayil, and A.G. Alleyne, “A survey of iterative learning control: A learning-based method for high-performance tracking control”, *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[2] Longman, R.W., “Iterative learning control and repetitive control for engineering practice”, *International Journal of Control*, vol. 73, no. 10, pp. 930–954, 2000.